<u>6-3 Square Root Functions and Inequalities</u>

Identify the domain and range of each function.

1. $f(x) = \sqrt{4x}$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.

$$\text{D} = \{x \mid x \geq 0\}$$
$$\text{R} = \{f(x) \mid f(x) \geq 0\}$$

2. $f(x) = \sqrt{x-5}$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.

$$\text{D} = \{x \mid x \geq 5\}$$
$$\text{R} = \{f(x) \mid f(x) \geq 0\}$$

3. $f(x) = \sqrt{x+8} - 2$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.

So:

$$x + 8 \geq 0$$
$$x \geq -8$$
$$\text{D} = \{x \mid x \geq -8\}$$

Find $f(-8)$ to determine the lower limit of the range.

$$f(-2) = \sqrt{-8+8} - 2$$
$$= -2$$

$$\text{R} = \{f(x) \mid f(x) \geq -2\}$$

Graph each function. State the domain and range.

4. $f(x)=\sqrt{x}-2$

SOLUTION:

The domain of a square root function only includes values for which the radicand is nonnegative.

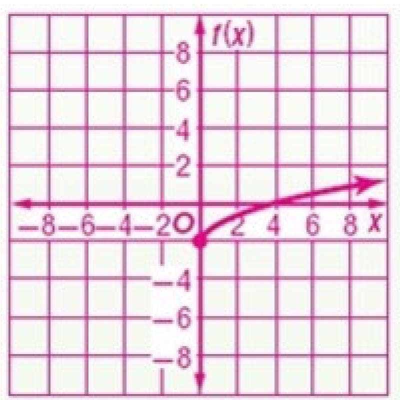
$D=\{x \mid x \geq 0\}$

Find $f(0)$ to determine the lower limit of the range.

$f(0)=-2$

Therefore:

$R=\{f(x) \mid f(x) \geq -2\}$



5. $f(x) = 3\sqrt{x-1}$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.

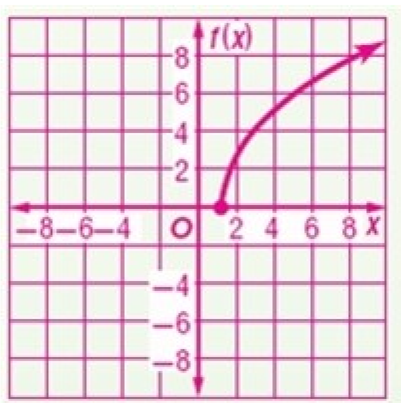
So:
$$x - 1 \geq 0$$
$$x \geq 1$$

$$D = \{x \mid x \geq 1\}$$

Find $f(1)$ to determine the lower limit of the range.

$$f(1) = 0$$

$$R = \{f(x) \mid f(x) \geq 0\}$$



6. $f(x)=\frac{1}{2}\sqrt{x+4}-1$

SOLUTION:

The domain of a square root function only includes values for which the radicand is nonnegative.

So:

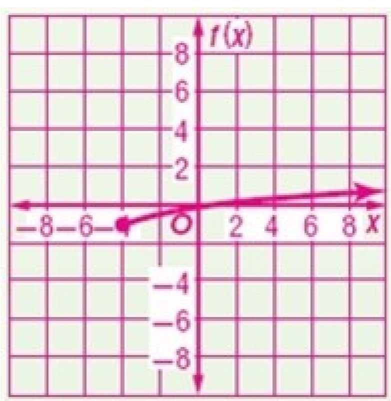
$$\begin{aligned} x+4 &\geq 0 \\ x &\geq -4 \end{aligned}$$

$D = \{x \mid x \geq -4\}$

Find $f(-4)$ to determine the lower limit of the range.

$$\begin{aligned} f(-4) &= \frac{1}{2}\sqrt{-4+4}-1 \\ &= -1 \end{aligned}$$

$R = \{f(x) \mid f(x) \geq -1\}$



6-3 Square Root Functions and Inequalities

7. $f(x) = -\sqrt{3x - 5} + 5$

SOLUTION:

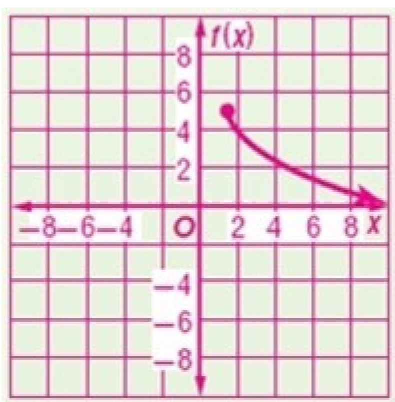
The domain of a square root function only includes values for which the radicand is nonnegative.

$$3x - 5 \geq 0$$
$$x \geq \frac{5}{3}$$

$$\mathrm{D} = \left\{x \middle| x \geq \frac{5}{3}\right\}$$

Find $f\left(\frac{5}{3}\right)$ to determine the upper limit of the range.

$$\mathrm{R} = \{f(x) | f(x) \leq 5\}$$



8. **OCEAN** The speed that a tsunami, or tidal wave, can travel is modeled by the equation $v = 356\sqrt{d}$, where v is the speed in kilometers per hour and d is the average depth of the water in kilometers. A tsunami is found to be traveling at 145 kilometers per hour. What is the average depth of the water? Round to the nearest hundredth of a kilometer.

SOLUTION:

$v = 356\sqrt{d}$

Substitute v = 145 and find d.

$$\begin{aligned} 145 &= 356\sqrt{d} \\ \sqrt{d} &= \frac{145}{356} \\ d &\approx 0.17\,\text{km} \end{aligned}$$

The average depth of the water is about 0.17 km.

Graph each inequality.

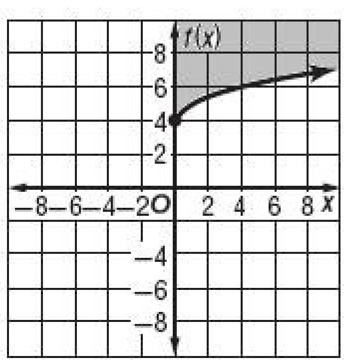
9. $f(x) \geq \sqrt{x} + 4$

SOLUTION:

$\text{D} = \{x \mid x \geq 0\}$

$\text{R} = \{y \mid y \geq 4\}$

Graph the $f(x) \geq \sqrt{x} + 4$.



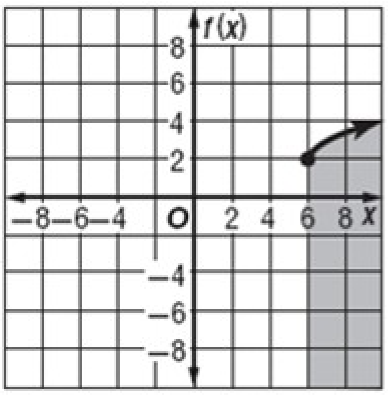
10. $f(x) \le \sqrt{x-6} + 2$

SOLUTION:

$\mathrm{D} = \{x \mid x \ge 6\}$

$\mathrm{R} = \{y \mid y \ge 2\}$

Graph the inequality $f(x) \le \sqrt{x-6} + 2$.



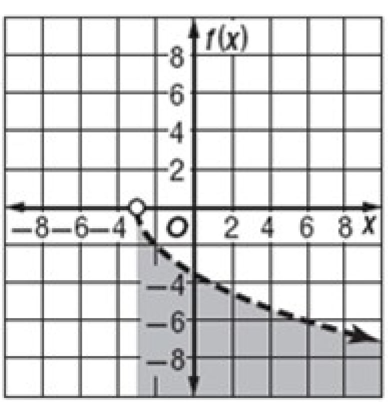
11. $f(x) < -2\sqrt{x+3}$

SOLUTION:

$\mathrm{D} = \{x \mid x \ge -3\}$

$\mathrm{R} = \{y \mid y < 0\}$

Graph the inequality $f(x) < -2\sqrt{x+3}$.

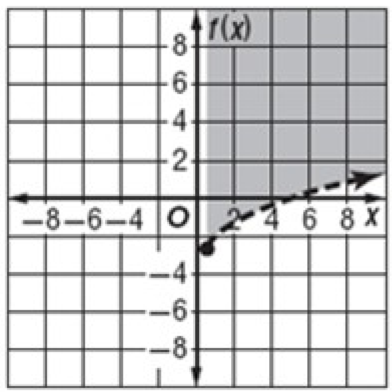


12. $f(x) > \sqrt{2x-1} - 3$

SOLUTION:

$D = \left\{x \middle| x \geq \frac{1}{2}\right\}$

$R = \{y | y > -3\}$



Identify the domain and range of each function.

13. $f(x) = -\sqrt{2x} + 2$

SOLUTION:

The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x | x \geq 0\}$

Find $f(0)$ to determine the upper limit of the range.

$R = \{f(x) | f(x) \leq 2\}$

14. $f(x)=\sqrt{x}-6$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x \mid x \geq 0\}$

$R = \{f(x) \mid f(x) \geq -6\}$

15. $f(x)=4\sqrt{x-2}-8$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x \mid x \geq 2\}$

Find $f(2)$ to determine the lower limit of the range.

$f(2)=-8$

$R = \{f(x) \mid f(x) \geq -8\}$

16. $f(x)=\sqrt{x+2}+5$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x \mid x \geq -2\}$

Find $f(-2)$ to determine the lower limit of the range.

$f(-2)=5$

$R = \{f(x) \mid f(x) \geq 5\}$

17. $f(x) = \sqrt{x-4} - 6$

SOLUTION:

The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x \mid x \geq 4\}$

Find $f(4)$ to determine the lower limit of the range.

$R = \{f(x) \mid f(x) \geq -6\}$

18. $f(x) = -\sqrt{x-6} + 5$

SOLUTION:

The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x \mid x \geq 6\}$

Find $f(6)$ to determine the upper limit of the range.

$R = \{f(x) \mid f(x) \leq 5\}$

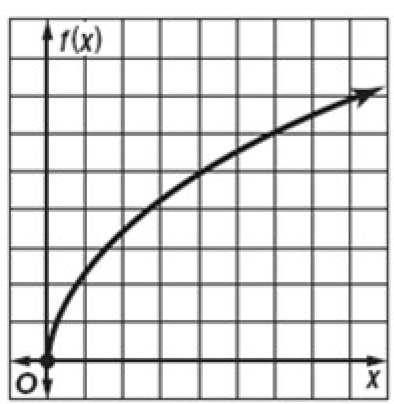
Graph each function. State the domain and range.

19. $f(x) = \sqrt{6x}$

SOLUTION:

$D = \{x \mid x \geq 0\}$

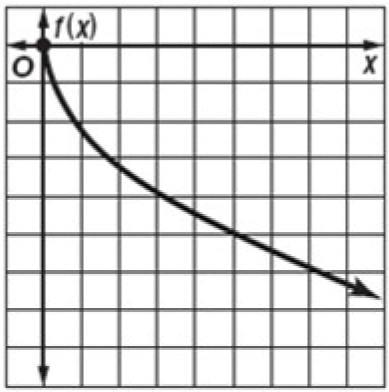
$R = \{f(x) \mid f(x) \geq 0\}$



20. $f(x) = -\sqrt{5x}$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.

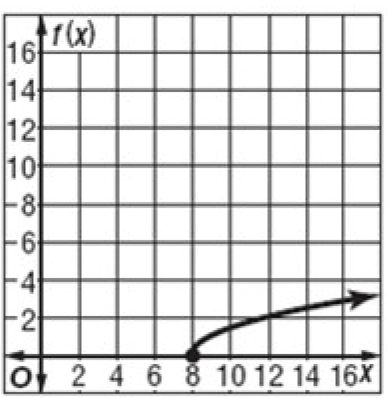
$D = \{x \mid x \geq 0\}$
$R = \{f(x) \mid f(x) \leq 0\}$



21. $f(x) = \sqrt{x-8}$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x \mid x \geq 8\}$
$R = \{f(x) \mid f(x) \geq 0\}$



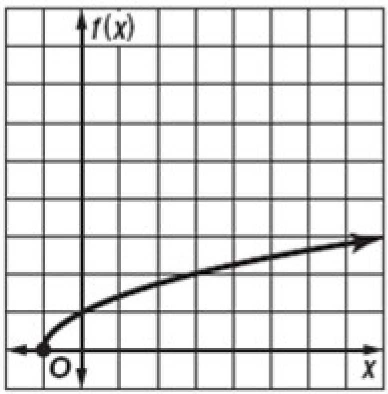
22. $f(x) = \sqrt{x+1}$

SOLUTION:

The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x | x \geq -1\}$

$R = \{f(x) | f(x) \geq 0\}$



23. $f(x) = \sqrt{x+3} + 2$

SOLUTION:

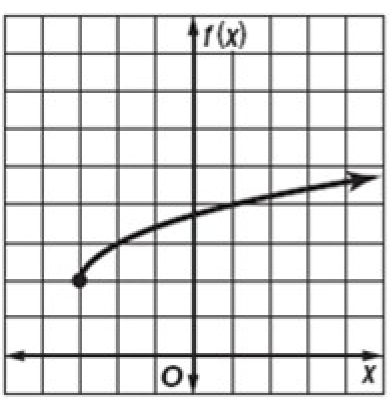
The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x | x \geq -3\}$

Find $f(-3)$ to determine the lower limit of the range.

$f(-3) = 2$

$R = \{f(x) | f(x) \geq 2\}$



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24. $f(x) = \sqrt{x-4} - 10$

SOLUTION:

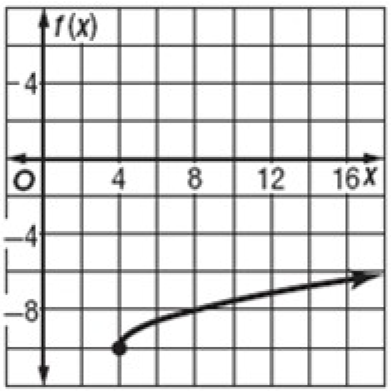
The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x | x \geq 4\}$

Find $f(4)$ to determine the lower limit of the range.

$f(4) = -10$

$R = \{f(x) | f(x) \geq -10\}$



25. $f(x) = 2\sqrt{x-5} - 6$

SOLUTION:

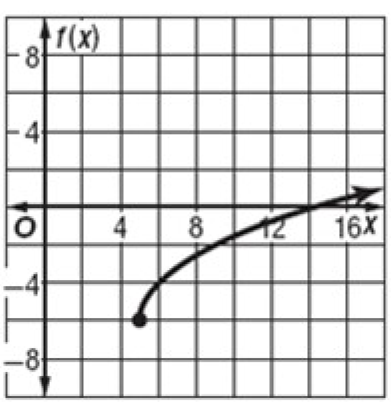
The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x | x \geq 5\}$

Find $f(5)$ to determine the lower limit of the range.

$f(4) = -6$

$R = \{f(x) | f(x) \geq -6\}$



6-3 Square Root Functions and Inequalities

26. $f(x) = \frac{3}{4}\sqrt{x+12} + 3$

SOLUTION:

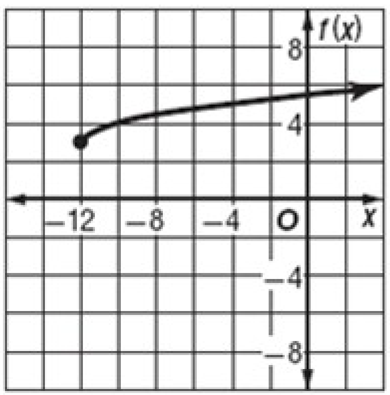
The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x | x \geq -12\}$

Find $f(-12)$ to determine the lower limit of the range.

$f(4) = 3$

$R = \{f(x) | f(x) \geq 3\}$



27. $f(x) = -\frac{1}{5}\sqrt{x-1} - 4$

SOLUTION:

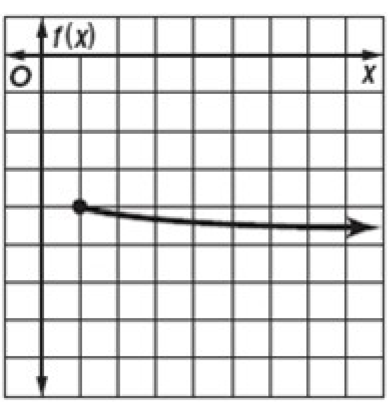
The domain of a square root function only includes values for which the radicand is nonnegative.

$D = \{x | x \geq 1\}$

Find $f(1)$ to determine the upper limit of the range.

$f(4) = -4$

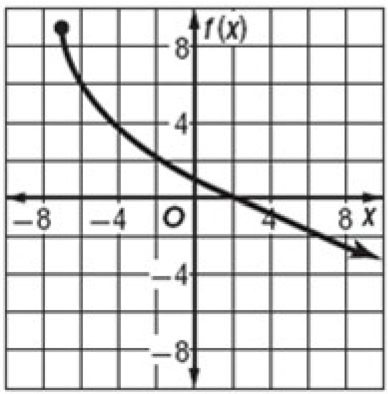
$R = \{f(x) | f(x) \leq -4\}$



28. $f(x) = -3\sqrt{x+7} + 9$

SOLUTION:
The domain of a square root function only includes values for which the radicand is nonnegative.
$D = \{x \mid x \geq -7\}$

Find $f(1)$ to determine the upper limit of the range.
$f(4) = 9$
$R = \{f(x) \mid f(x) \leq 9\}$



29. **SKYDIVING** The approximate time t in seconds that it takes an object to fall a distance of d feet is given by $t = \sqrt{\frac{d}{16}}$. Suppose a parachutist falls 11 seconds before the parachute opens. How far does the parachutist fall during this time?

SOLUTION:
Substitute $t = 11$ in the equation and find d.

$$11 = \sqrt{\frac{d}{16}}$$
$$121 = \frac{d}{16}$$
$$d = 121(16)$$
$$d = 1936$$

The parachutist falls 1936 feet.

30. **CCSS MODELING** The velocity of a roller coaster as it moves down a hill is $V = \sqrt{v^2 + 64h}$, where v is the initial velocity in feet per second and h is the vertical drop in feet. The designer wants the coaster to have a velocity of 90 feet per second when it reaches the bottom of the hill.

a. If the initial velocity of the coaster at the top of the hill is 10 feet per second, write an equation that models the situation.

b. How high should the designer make the hill?

SOLUTION:

a. Substitute v = 10 and V = 90.

$90 = \sqrt{100 + 64h}$

b. Solve the equation $90 = \sqrt{100 + 64h}$ for h.

$$\begin{aligned} 90 &= \sqrt{100 + 64h} \\ 8100 &= 100 + 64h \\ 64h &= 8000 \\ h &= 125 \end{aligned}$$

The designer should make the hill at a height of 125 feet.

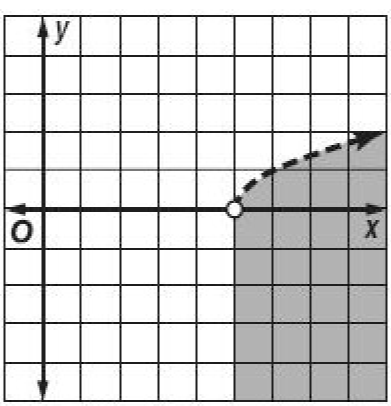
Graph each inequality.

31. $y < \sqrt{x - 5}$

SOLUTION:

Graph the boundary $y = \sqrt{x - 5}$. Since the inequality symbol is <, the boundary should be dashed.

The domain is $\{x \mid x \geq 5\}$. Because y is *less than*, the shaded region should be below the boundary and within the domain.

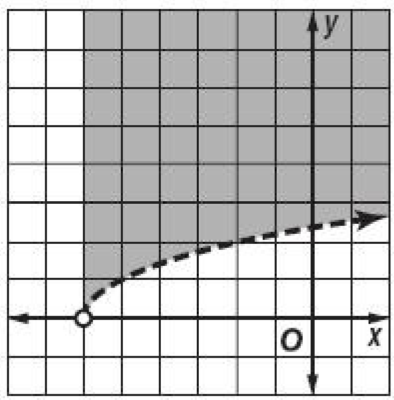


32. $y > \sqrt{x+6}$

SOLUTION:

Graph the boundary $y = \sqrt{x+6}$. Since the inequality symbol is >, the boundary should be dashed.

The domain is $\{x \mid x \geq -6\}$. Because y is *greater than*, the shaded region should be above the boundary and within the domain.

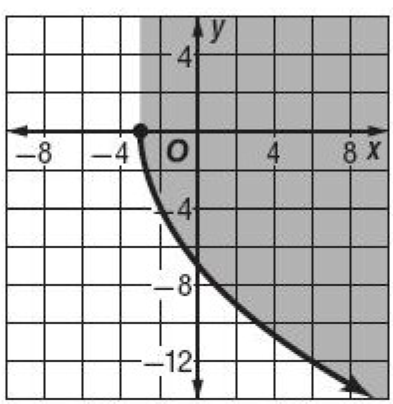


33. $y \geq -4\sqrt{x+3}$

SOLUTION:

Graph the boundary $y = -4\sqrt{x+3}$. Since the inequality symbol is $\geq$, the boundary line should be solid.

The domain is $\{x \mid x \geq -3\}$. Because y is *greater than*, the shaded region should be above the boundary and within the domain.

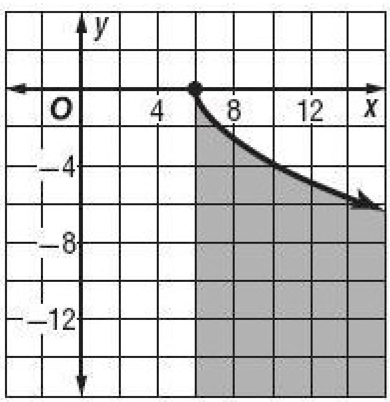


34. $y \le -2\sqrt{x-6}$

SOLUTION:

Graph the boundary $y = -2\sqrt{x-6}$. Since the inequality symbol is $\le$, the boundary line should be solid.

The domain is $\{x \mid x \ge 6\}$. Because y is *less than*, the shaded region should be below the boundary and within the domain.

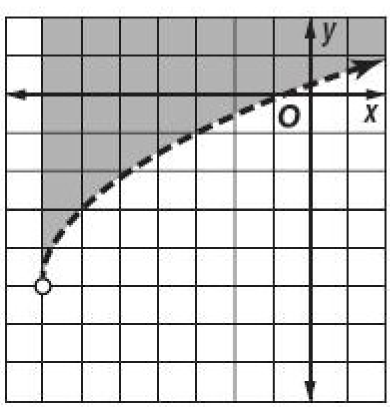


35. $y > 2\sqrt{x+7} - 5$

SOLUTION:

Graph the boundary $y = 2\sqrt{x+7} - 5$. Since the inequality symbol is >, the boundary line should be dashed.

The domain is $\{x \mid x \ge -7\}$. Because y is *greater than*, the shaded region should be above the boundary and within the range.

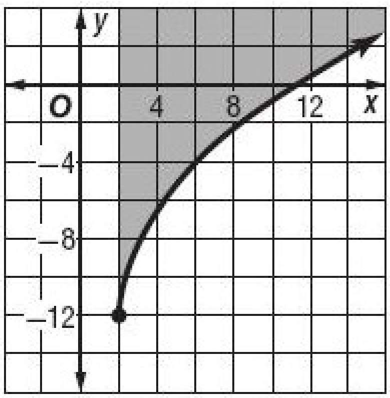


36. $y \geq 4\sqrt{x-2} - 12$

SOLUTION:

Graph the boundary $y = 4\sqrt{x-2} - 12$. Since the inequality symbol is $\geq$, the boundary line should be solid.

The domain is $\{x \mid x \geq 2\}$. Because y is *greater than*, the shaded region should be above the boundary and within the range.

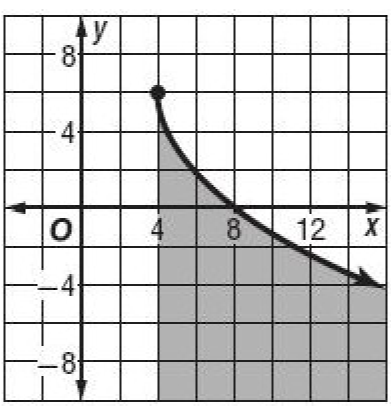


37. $y \leq 6 - 3\sqrt{x-4}$

SOLUTION:

Graph the boundary $y = 6 - 3\sqrt{x-4}$. Since the inequality symbol is $\leq$, the boundary line should be solid.

The domain is $\{x \mid x \geq 4\}$. Because y is *less than*, the shaded region should be below the boundary an
d within the range.

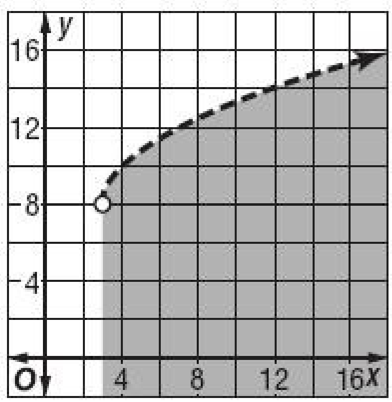


38. $y < \sqrt{4x - 12} + 8$

SOLUTION:

Graph the boundary $y = \sqrt{4x - 12} + 8$. Since the inequality symbol is <, the boundary line should be dashed.

The domain is $\{x | x \geq 3\}$. Because y is *less than*, the shaded region should be below the boundary and within the range.



39. **PHYSICS** The kinetic energy of an object is the energy produced due to its motion and mass. The formula for kinetic energy, measured in joules j, is $E = 0.5mv^2$, where m is the mass in kilograms and v is the velocity of the object in meters per second.

a. Solve the above formula for v.

b. If a 1500-kilogram vehicle is generating 1 million joules of kinetic energy, how fast is it traveling?

c. *Escape velocity* is the minimum velocity at which an object must travel to escape the gravitational field of a planet or other object.

Suppose a 100,000-kilogram ship must have a kinetic energy of 3.624×10^{14} joules to escape the gravitational field of Jupiter. Estimate the escape velocity of Jupiter.

SOLUTION:

a. $E = 0.5mv^2$

Solve for v.

$$E = \frac{1}{2}mv^2$$

$$\frac{2E}{m} = v^2$$

$$\sqrt{\frac{2E}{m}} = v$$

b. Substitute E = 1,000,000 and m = 1500. Find v.

$$v = \sqrt{\frac{2,000,000}{1500}}$$
$$\approx 36.5 \text{ m/s}$$

c. Substitute $E = 3.624 \times 10^{14}$ and m = 100,000. Find v.

$$v = \sqrt{\frac{7.248 \times 10^{14}}{100,000}}$$
$$\approx 85135 \text{ m/s}$$

40. **CCSS REASONING** After an accident, police can determine how fast a car was traveling before the driver put on his or her brakes by using the equation $v = \sqrt{30fd}$. In this equation, v represents the speed in miles per hour, f represents the coefficient of friction, and d represents the length of the skid marks in feet. The coefficient of friction varies depending on road conditions. Assume that $f = 0.6$.

a. Find the speed of a car that skids 25 feet.

b. If your car is going 35 miles per hour, how many feet would it take you to stop?

c. If the speed of a car is doubled, will the skid be twice as long? Explain.

SOLUTION:

a. Substitute $f = 0.6$ and $d = 25$. Find v.

$$v = \sqrt{30 \times 0.6 \times 25} \approx 21.2 \text{ mph}$$

$$v = \sqrt{30fd}$$

b. $v^2 = 30fd$

$$d = \frac{v^2}{30f}$$

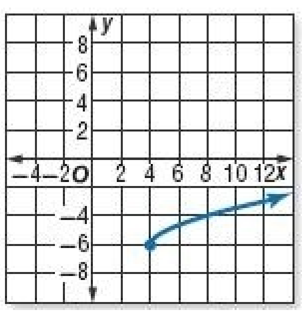
Substitute $v = 35$ and $f = 0.6$.

$$d = \frac{35^2}{30 \times 0.6} \approx 68 \text{ ft}$$

c. No; it is not a linear function. The skid will be 4 times as long.

Write the square root function represented by each graph.

41.



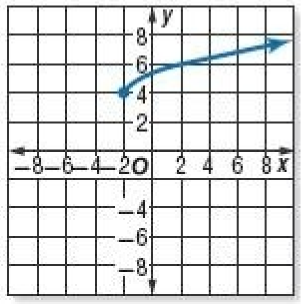
SOLUTION:

$D = \{x \mid x \geq 4\}$

$R = \{y \mid y \geq -6\}$

The graph represents the function $y = \sqrt{x-4} - 6$.

42.



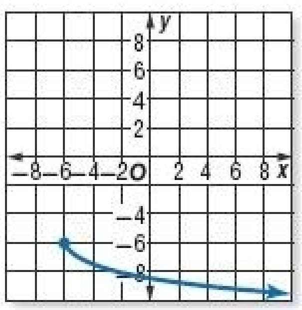
SOLUTION:

$D = \{x | x \geq -2\}$

$R = \{y | y \geq 4\}$

The graph represents the function $y = \sqrt{x + 2} + 4$.

43.



SOLUTION:

$D = \{x | x \geq -6\}$

$R = \{y | y \leq -6\}$

The graph represent the function $y = -\sqrt{x + 6} - 6$.

44. **MULTIPLE REPRESENTATIONS** In this problem, you will use the following functions to investigate transformations of square root functions.

$f(x) = 4\sqrt{x - 6} + 3$	$g(x) = \sqrt{16x + 1} - 6$	$h(x) = \sqrt{x + 3} + 2$

a. GRAPHICAL Graph each function on the same set of axes.

b. ANALYTICAL Identify the transformation on the graph of the parent function. What values caused each transformation?

c. ANALYTICAL Which functions appear to be stretched or compressed vertically? Explain your reasoning.

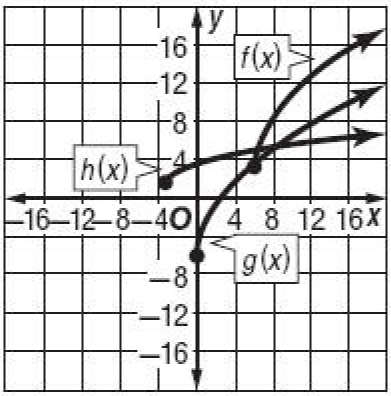
d. VERBAL The two functions that are stretched appear to be stretched by the same magnitude. How is this

possible?

e. TABULAR Make a table of the rate of change for all three functions between 8 and 12 as compared to 12 and 16. What generalization about rate of change in square root functions can be made as a result of your findings?

SOLUTION:

a.



b. $f(x)$: 6 units to the right, 3 units up; $g(x)$: $\frac{1}{16}$ to the left, 6 units down; $h(x)$: 3 units to the left, 2 units up.

c. Sample answer: $f(x)$ and $g(x)$ appear to be stretched because the graph increases much more quickly than the parent graph.

d. Sample answer: They are stretched by the same magnitude because $4 = \sqrt{16}$.

e.

$x = 8$	$x = 12$	$x = 16$	Rate of Change between 8 and 12	Rate of Change between 12 and 16
$f(8) = 8.66$	$f(12) = 12.798$	$f(16) = 15.65$	1.0345	0.713
$g(8) = 5.36$	$g(12) = 7.89$	$g(16) = 10.03$	0.6325	0.535
$h(8) = 5.317$	$h(12) = 5.873$	$h(16) = 6.359$	0.139	0.1215

The rate of change decreases as the x-values increase for square root functions of the form $a\sqrt{x-b}+c$, where $a > 0$.

45. **PENDULUMS** The period of a pendulum can be represented by $T = 2\pi\sqrt{\frac{L}{g}}$, where T is the time in seconds, L is the length in feet, and g is gravity, 32 feet per second squared.

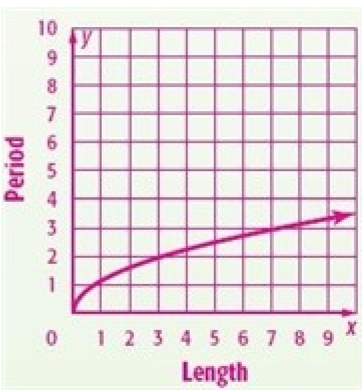
a. Graph the function for $0 \le L \le 10$.

b. What is the period for lengths of 2, 5, and 8 feet?

SOLUTION:

a. Substitute 32 for g in $T = 2\pi\sqrt{\frac{L}{g}}$.

$$T = 2\pi\sqrt{\frac{L}{32}}$$



b. $T = 2\pi\sqrt{\frac{L}{g}}$

Substitute L = 2 and g = 32.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{2}{32}} \\ &\approx 1.57 \end{aligned}$$

Substitute L = 5 and g = 32.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{5}{32}} \\ &\approx 2.48 \end{aligned}$$

Substitute L = 8 and g = 32.

$$T = 2\pi\sqrt{\frac{8}{32}}$$
$$\approx 3.14$$

46. **PHYSICS** Using the function $m = \frac{m_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$, Einstein's theory of relativity states that the apparent mass m of a particle depends on its velocity v. An object that is traveling extremely fast, close to the speed of light c, will appear to have more mass compared to its mass at rest, m_0.

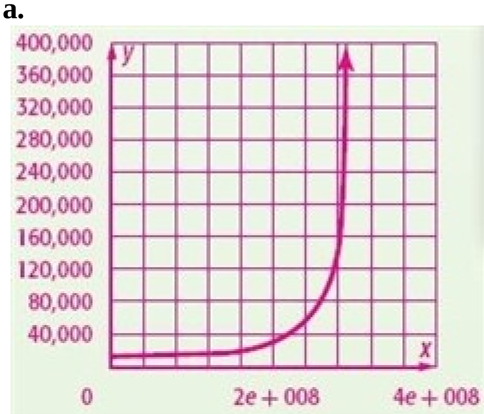
a. Use a graphing calculator to graph the function for a 10,000-kilogram ship for the domain $0 \le v \le 300{,}000{,}000$. Use 300 million meters per second for the speed of light.

b. What viewing window did you use to view the graph?

c. Determine the apparent mass m of the ship for speeds of 100 million, 200 million, and 299 million meters per second.

SOLUTION:

a.



b. {0, 400,000,000} scl: 50,000,000 by {0, 100,000} scl: 20,000

c. (100 million, 10,607)(200 million, 13,416)(299 million, 122,577)

47. **CHALLENGE** Write an equation for a square root function with a domain of $\{x \mid x \geq -4\}$, a range of $\{y \mid y \leq 6\}$, and that passes through $(5, 3)$.

SOLUTION:

Sample answer: Analyze the domain first. Since $x > -4$, let the radical expression be $\sqrt{x+4}$. According to the range given, the maximum y-value is 6. When x = -4, the radical expression is 0. Add 6 to this so that the range will be $y \leq 6$. Since the range states that the maximum value for y is 6 and the domain states that the minimum value for x is -4, the radical expression must be negative in order to satisfy these constraints.

$$y = -\sqrt{x+4} + 6$$

Check the point (5, 3).

$$y = -\sqrt{x+4} + 6$$
$$3 \stackrel{?}{=} -\sqrt{5+4} + 6$$
$$3 \stackrel{?}{=} -\sqrt{9} + 6$$
$$3 \stackrel{?}{=} -3 + 6$$
$$3 = 3$$

48. **REASONING** For what positive values of a are the domain and range of $f(x) = \sqrt[a]{x}$ the set of real numbers?

SOLUTION:
All positive odd numbers; the set of even numbers would result in a range of nonnegative values.

49. **OPEN ENDED** Write a square root function for which the domain is $\{x \mid x \geq 8\}$ and the range is $\{y \mid y \leq 14\}$.

SOLUTION:

Sample answer: Analyze the domain first. Since $x > 8$, let the radical expression be $\sqrt{x-8}$. According to the range given, the maximum y-value is 14. When x = 4, the radical expression is 0. Add 14 to this so that the range will be $y \leq 14$. Since the range states that the maximum value for y is 14 and the domain states that the minimum value for x is 8, the radical expression must be negative in order to satisfy these constraints.

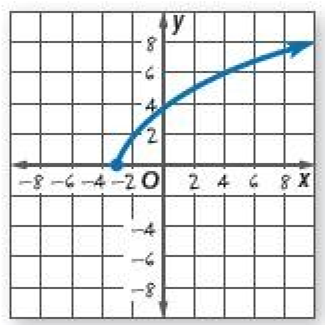
$$y = -\sqrt{x-8} + 14$$

6-3 Square Root Functions and Inequalities

50. **WRITING IN MATH** Explain why there are limitations on the domain and range of square root functions.

SOLUTION:
Sample answer: The domain is limited because square roots of negative numbers are imaginary. The range is limited due to the limitation of the domain.

51. **CCSS CRITIQUE** Cleveland thinks that the graph and the equation represent the same function. Molly disagrees. Who is correct? Explain your reasoning.



$y = \sqrt{5x + 10}$

SOLUTION:
$y = \sqrt{5x + 10}$ has an x-intercept of -2 and would be to the right of the given graph. So, Molly is correct.

52. **WRITING IN MATH** Explain why $y = \pm\sqrt{x}$ is not a function.

SOLUTION:
To be a function, for every x-value there must be exactly one y-value. For every x in this equation there are two y-values, one that is negative and one that is positive. Also, the graph of $y = \pm\sqrt{x}$ does not pass the vertical line test.

53. **OPEN ENDED** Write an equation of a relation that contains a radical and its inverse such that:

a. the original relation is a function, and its inverse is not a function
.
b. the original relation is not a function, and its inverse is a function.

SOLUTION:
a. Sample answer: The original is $y = x^2 + 2$ and inverse is $y = \pm\sqrt{x - 2}$.

b. Sample answer: The original is $y = \pm\sqrt{x} + 4$ and inverse is $y = (x - 4)^2$.

54. The expression $\frac{-64x^6}{8x^3}, x \neq 0$, is equivalent to

A $8x^2$

B $8x^3$

C $-8x^2$

D $-8x^3$

SOLUTION:

$$\begin{aligned}\frac{-64x^6}{8x^3} &= -8x^{6-3} \\ &= -8x^3\end{aligned}$$

The correct choice is D.

55. **PROBABILITY** For a game, Patricia must roll a standard die and draw a card from a deck of 26 cards, each card having a letter of the alphabet on it. What is the probability that Patricia will roll an odd number and draw a letter in her name?

F $\frac{2}{3}$

G $\frac{3}{26}$

H $\frac{1}{13}$

J $\frac{1}{26}$

SOLUTION:
Let A be the event *getting an odd number on the die,* and B be the event *drawing a letter in her name*.

$P(A) = \frac{1}{2}$

$P(B) = \frac{6}{26}$

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B) \\ &= \frac{1}{2} \cdot \frac{6}{26} \\ &= \frac{3}{26} \end{aligned}$$

The correct choice is G.

56. **SHORT RESPONSE** What is the product of $(d + 6)$ and $(d - 3)$?

SOLUTION:

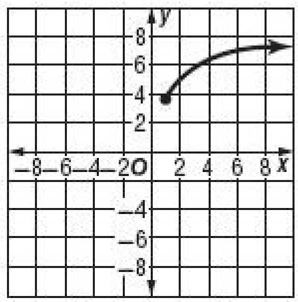
$$\begin{aligned} (d+6)(d-3) &= d^2 - 3d + 6d - 18 \\ &= d^2 + 3d - 18 \end{aligned}$$

57. **SAT/ACT** Given the graph of the square root function below, which must be true?

I. The domain is all real numbers.

II. The function is $y = \sqrt{x} + 3.5$.

III. The range is about $\{y \mid y \geq 3.5\}$.



A I only

B I, II, and III

C II and III only

D II only

E III only

SOLUTION:

The domain of the given function appears to be $x \geq 1$.
So statement I cannot be true.

Statement II can also not be true; the domain of $y = \sqrt{x} + 3.5$ is $x \geq 0$.
Statement II is true. So, the correct answer is E.

Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

58. $f(x) = 2x$
$g(x) = \frac{1}{2}x$

SOLUTION:

The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{1}{2}x\right) \\ &= 2\left(\frac{1}{2}x\right) \\ &= x\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(2x) \\ &= \frac{1}{2}(2x) \\ &= x\end{aligned}$$

Therefore, f and g are inverses.

59. $f(x) = 3x - 7$
$g(x) = \frac{1}{3}x - \frac{7}{16}$

SOLUTION:

The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{1}{3}x - \frac{7}{16}\right) \\ &= 3\left(\frac{1}{3}x - \frac{7}{16}\right) - 7\end{aligned}$$

Since $[f \circ g](x) \neq x$, f and g are not inverses.

60. $$f(x) = \frac{3x+2}{5}$$
$$g(x) = \frac{5x-2}{3}$$

SOLUTION:

The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{5x-2}{3}\right) \\ &= \frac{3\left(\frac{5x-2}{3}\right)+2}{5} \\ &= \frac{5x-2+2}{5} \\ &= x\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g\left(\frac{3x+2}{5}\right) \\ &= \frac{5\left(\frac{3x+2}{5}\right)-2}{3} \\ &= x\end{aligned}$$

Therefore, f and g are inverses.

61. **TIME** The formula $h = \frac{m}{60}$ converts minutes m to hours h, and $d = \frac{h}{24}$ converts hours h to days d. Write a function that converts minutes to days.

SOLUTION:

$$\begin{aligned}d(h) &= \frac{h}{24} \\ [d \circ h](m) &= d[h(m)] \\ &= d\left(\frac{m}{60}\right) \\ &= \frac{\left(\frac{m}{60}\right)}{24} \\ &= \frac{m}{1440}\end{aligned}$$

62. **CABLE TV** The number of households in the United States with cable TV after 1985 can be modeled by the function $C(t) = -43.2t^2 + 1343t + 790$, where t represents the number of years since 1985.

a. Graph this equation for the years 1985 to 2005.

b. Describe the turning points of the graph and its end behavior.

c. What is the domain of the function? Use the graph to estimate the range for the function.

d. What trends in households with cable TV does the graph suggest? Is it reasonable to assume that the trend will continue indefinitely?

SOLUTION:

a.

b. rel. max. between $t = 15$ and $t = 16$, and no rel. min.;
$C(t) \to -\infty$ as $t \to -\infty$, $C(t) \to -\infty$ as $t \to +\infty$

c. D = {all real numbers}; $R = \{y \mid y \le 11{,}225\}$

d. The number of cable TV systems rose steadily from 1985 to 2000. Then the number began to decline. The trend may continue for some years, but the number of cable TV systems cannot decline at this rate indefinitely. The number cannot fall below 0. It is not likely that the number would come close to 0 for the foreseeable future; there is no reason to believe that cable TV systems will not be in use.

Determine whether each number is rational or irrational.

63. 6.34

SOLUTION:

6.34 can be written in the form $\frac{p}{q}, q \neq 0$, for example, as follows:

$\frac{634}{100}$

Therefore, it is a rational number.

64. 3.787887888…

SOLUTION:
3.787887888… neither terminates nor repeats. Therefore, it is irrational.

65. 5.333…

SOLUTION:

The number can be written in the form $\frac{p}{q}, q \neq 0$, for example, as follows:

$\frac{16}{3}$

Therefore, it is a rational number.

66. 1.25

SOLUTION:

The number can be written in the form $\frac{p}{q}, q \neq 0$, for example, as follows:

$\frac{5}{4}$

Therefore, it is a rational number.